Bachelor of Science (B.Sc.) Semester—II (C.B.S.) Examination

MATHEMATICS

Compulsory Paper—2

(M_4 Vector Calculus and Improper Integrals)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the velocity and acceleration of a particle which moves along the curve x = 2 sin 3t, y = 2 cos 3t, z = 8t at any time t > 0. Also find the magnitude of the velocity and acceleration. 6

(B) If $\bar{A} = \frac{\bar{r}}{r}$, where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then show that $\nabla \circ \bar{A} = \frac{2}{r}$ and $\nabla(\nabla \circ \bar{A}) = -2\frac{\bar{r}}{r^3}$. 6

OR

(C) Show that $\bar{A} = \frac{\bar{r}}{r^2}$ is irrotational, where $\bar{r} = x\bar{i} + y\bar{j} + 2\bar{k}$. 6

(D) Find the work done by the force field $\bar{F} = (5xy - 6x^2)\,\bar{i} + (2y - 4x)\,\bar{j}$ in moving a particle along the curve C : $y = x^3$ in the xy-plane from the point (1, 1) to (2, 8). 6

UNIT—II

2. (A) Find the area of region lying between the parabola $y = x^2$ and the line y = x. 6

(B) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dydx$ by changing the order of integration. 6

OR

(C) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} y^2\sqrt{x^2 + y^2}dxdy$ by changing to polar coordinates. 6

(D) Evaluate $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x^2 dzdydx$. 6

UNIT—III

3. (A) Evaluate $\iiint_V (2x+y)dV$, where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 2$, $z = 0$. 6

(B) Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = 0$, $y = 0$, $x + y = 1$. 6

OR

(C) Verify Stoke's theorem when $\bar{F} = (2x - y)\bar{i} - yz^2\bar{j} - y^2z\bar{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 6

(D) Evaluate $\iint_S \bar{A} \circ \bar{n}\, ds$ where $\bar{A} = (x + y^2)\,\bar{i} - 2x\,\bar{j} + 2yz\,\bar{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. 6

UNIT—IV

4. (A) If $\int_a^{\infty} |f(x)|\,dx$ converges, then prove that $\int_a^{\infty} f(x)dx$ converges. Hence prove that $\int_0^{\infty} \frac{\cos x}{x^2+1}dx$ is convergent. 6

(B) Test the convergence of :—

(i) $\int_1^5 \frac{dx}{\sqrt{x^4 - 1}}$

(ii) $\int_0^3 \frac{dx}{(3-x)\sqrt{x^2+1}}$. 6

OR

(C) Prove that :

(i) $\Gamma(n+1) = n\Gamma n$, $n > 0$ and

(ii) $\int_0^{\infty} x^{n-1}e^{-kx}dx = \frac{\Gamma n}{k^n}$. 6

(D) Show that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}dx$. 6

Question—V

5. (A) Show that the vector :

$\overline{V} = (x + 3y)\,\overline{i} + (y - 2z)\overline{j} + (x - 2z)\overline{k}$ is solenoidal. 1½

(B) If $\overline{R} = \sin t\,\overline{i} + \cos t\,\overline{j} + t\overline{k}$, then find $\frac{d\overline{R}}{dt}$ and $\left|\frac{d\overline{R}}{dt}\right|$. 1½

(C) Evaluate $\int_0^{\pi/2}\int_0^{\cos\theta} r\,dr d\theta$. 1½

(D) Evaluate $\int_0^1\int_0^2\int_0^3 (x + y + z)\,dz\,dy\,dx$. 1½

(E) Express Green's theorem in the plane in vector notation. 1½

(F) If $\overline{H}$ = curl $\overline{A}$, then show that $\iint_S \overline{H} \circ \overline{n}\,ds = 0$ for any closed surface S by using divergence theorem. 1½

(G) Test the convergence of $\int_1^{\infty} \frac{x dx}{2x^3 + 3x + 5}$ by comparison test. 1½

(H) Evaluate $\int_0^{\pi/2} \sin^2\theta \cos^3\theta\, d\theta$. 1½